DYNAMIC BI-PRODUCT BUNDLE PRICING PROBLEM

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This paper addresses bundle pricing problem of two products in a stochastic environment so as to maximize net profit of a retailer. In the considered problem, it is assumed that customers are received upon a Poisson distribution and their demands follow a bi-variant distribution function. Also, it is assumed that products are sold individually or in the form of a bundle, which are offered from an initial stock of the products. To tackle the problem, a stochastic dynamic program is developed in which optimum values of the initial stock and order quantities of every planning period are determined. Moreover, prices of the individual products and their bundle are optimized. Also, the proposed dynamic program tackles bundling/ unbundling decisions taken in every planning period. A numerical example of a two planning period horizon is considered to validate the proposed model.

Key words: bundle pricing, marketing, stochastic dynamic programming, product bundling

INTRODUCTION

The most prevailing aims of any firms delivering products or services to the customers are profit maximization which has been mainly achieved by means of cost reduction. Despite of past decades, other disciplines have been recently adopted rather than cost reduction methods, among which pricing as a method of demand management might be of special interest to the academic and practical societies. As categorized in (Roth, 2007), several pricing objectives are considered, such as marginal profit maximization, revenue maximization, market share maximization and status quo, each of which requires suitable pricing strategy. Bundle pricing is one of the pricing strategies which have been applied successfully in diverse fields of industry and services. By adopting bundle pricing strategy, several products/ services are offered to the customers in a single package for a pre-determined price (Kinberg and Sudit, 1979).

From customer's point of view, a product is bought when the relevant consumer surplus is positive. In other words, customer buys a product when it is worthy enough to pay the label price. The highest level of price which is desirable to the customer is called reservation price (RP) or maximum willingness to pay. Hence, if reservation price of a product is greater than its label price, the customer buys the product (Jedidi and Zhang, 2002). In the case of bundling, it is important what relation the bundle components have with each other. In this regard, three cases are possible; complementarity, substitution, and independency. Complement components are the ones whose bundle's RP is greater than sum of their RPs; whilst the opposite case refers to substitute products (RPbundle<RP1+RP2). The third case corresponds to independent components for which bundle RP equals to sum of their RPs (Venkatesh and Kamakura, 2003). In this paper, pricing and bundling decisions of two products with stochastic demands are addressed. In this regard, it is decided what quantities of products are ordered to meet demands of individual products and their bundled form. In the considered problem, incoming customers might buy (a) an individual product, or (b) bundle of products, or (c) nothing. Also, shortage is allowed for any of products and hence,

the bundle. In the case of any shortages, customers might choose to wait for their desirable purchase until the next period. In this case, the seller is charged with the cost of backordering. Remainder of the paper is organized as follows. Section 2 reviews literature body of the problem briefly, while the developed stochastic dynamic program is explained in Section 3. Section 4 reports the conducted experimental results and Section 5 concludes the paper with concluding remarks and future research directions.

LITERATURE REVIEW

A limited number of published papers in the field of bundle pricing is related to the ones which have adopted dynamic programming approach. Aydin and Ziya (2008) addressed upselling using dynamic programming. In their considered upselling problem, a regular product was bundled with a promotional one, while the promotional product was sold at a discounted price (sub-additive bundle prices). Ferrer et al. (2010) considered bundling of a product with two alternative service levels (two substitute bundles). They developed a dynamic program to maximize seller's profit. Also, they assumed no bundling cost and switching cost of customer from one bundle to the other one. Another dynamic program is found in (Ferreira and Wu, 2011). The authors utilized an integrated approach for the bundling problem of a number of products using Data Envelopment Analysis (DEA) and Markovian processes. In the developed model, DEA was first used to determine the most efficient product bundles; then, a dynamic program was developed to determine prices of the selected efficient bundles. The authors defined product inventory levels as the state variables and they assumed no bundling cost and no disposal cost of unwanted products within the bundles in the developed model.

Two similar models to that of this paper are the models published in (Gürler et al., 2009; Bulut et al., 2009) to cope with joint pricing-inventory problem. The addressed problem is related to a retailer with two perishable products with different details. However, optimal prices of the products are determined with respect to the initial inventory and the customer order rate in a Poisson process. In these papers, customers have three choices; buying bundle of two products, buying one of the two products, and buying nothing. In this paper, a stochastic dynamic program is proposed to tackle pricing decisions of two products which are sold either individually or as bundle. In the developed model, it is assumed that customer might choose one of the products or their bundle or buying nothing. Also, shortage is allowed for the individual products or the bundle. In the case of shortage, customers might switch to the other product or wait until the next period (with a backorder cost for the seller).

MATHEMATICAL MODEL

In this section, an initial stock of two substitute/ complement products is assumed at the beginning of the planning horizon. Customers are received upon a Poisson process with average λ, who might select one of the products or their bundle. In the case of shortage, customer might switch to the other product or buy nothing. Moreover, one might wait until beginning of the next planning period to buy her desired product with a lower price. The proposed model seeks to optimize initial stock of the products at the beginning of the horizon as well as the quoted prices of the components and their bundle. Also, bundling/ unbundling decisions are made at the beginning of the planning period. Followings present assumptions of the considered problem upon which the mathematical model is formulated. Nomenclature and the formulation are proposed afterwards.

i	Index of product $(i = 1, 2, b)$; b stands for the bundle of products
j	Index of time period
k	Index of switched product $(k = 1, 2, b)$
ch_i	Unit holding cost of product i per period
c_i	Unit cost of product i $(i = 1, 2)$
$c_{bundling}$	Bundling cost
$c_{unbundling}$	Unbundling cost
cb_i	Unit backordering cost of product i per period
λ	Arrival rate of customers per period

b_i	Threshold of consumer surplus with respect to the product i upon which it is decided whether customers wait to get the backordered product of the first period
R_i	Reservation price of product i
pi_i^j	Potential inventory of product i at the beginning of period j
I_i^j	Inventory level of product i at the end of period j
S_i^j	Inventory level of product i after ordering and before bundling/ unbundling at the beginning of period j
N_i^j	Inventory level of product i after bundling/ unbundling at the beginning of period j
n_i^j	Purchased units of product i in period j
$P(n_1^j, n_2^j, n_b^j, p_i^j)$	Selling probability of n_1^j units of Product 1, n_2^j units of Product 2, and n_b^j units of bundled products in period j, providing that product i is sold at price p_i
m_0	Probability that customers buy nothing
m_1	Probability that customers buy Product 1
m_2	Probability that customers buy Product 2
m_b	Probability that customers buy bundle of the products
γ_{ii}	Probability that customer wait for product i until the next period
γ_{ik}	Probability of customer switching from product i to product k $(k \neq i)$ in the case of product i's shortage
γ_{i0}	Probability of leaving without purchasing in the case of product i's shortage
x_i^j	Number of customers whose first preferences are product i at period j
x_{ii}^j	Numbers of customer waiting for product i at period 1 until the next period
x_{ik}^j	Number of customers who switched from product i to product k $(k \neq i)$ in period j
x_{i0}^j	Number of customers whose first preferences are product i at period j and they leave without any purchase
B_i^j	Backorder level of product i at the end of period j
l_i	Arrival rate of customers for product i
p_i^j	Price of product i at period j
Q_i^j	Number of ordered units of product i $(i = 1, 2)$ at the beginning of period j
$n_{bundling}^j$	Number of bundled products in period j
$n_{unbundling}^j$	Number of unbundled products in period j

$$V_j\left(pi_1^j, pi_2^j, pi_b^j\right) = \max_{\substack{p_i^j, Q_i^j, \\ n_{bundling}^j, n_{unbundling}^j}} \left\{ \begin{pmatrix} \sum_{n_1^j=0}^{N_1^j} \sum_{n_2^j=0}^{N_2^j} \sum_{n_b^j=0}^{N_b^j} P\left(n_1^j, n_2^j, n_b^j, p_i^j\right) \times \begin{pmatrix} p_1^j\left(n_1^j\right) + B_1^j\left(p_1^j - cb_1\right) + \\ p_2^j\left(n_2^j\right) + B_2^j\left(p_2^j - cb_2\right) + \\ p_b^j\left(n_b^j\right) + B_b^j\left(p_b^j - cb_b\right) - \\ ch_1 I_1^j - ch_2 I_2^j - ch_b I_b^j + \\ V_{j+1}\left(pi_1^{j+1}, pi_2^{j+1}, pi_b^{j+1}\right) \end{pmatrix} \\ -c_1 Q_1^j - c_2 Q_2^j - c_{unbundling} n_{unbundling}^j - c_{bundling} n_{bundling}^j \end{pmatrix} \right\}$$

Subject to

$$B_i^j = \begin{cases} 0 & E(x_{ii}^j) \le 0.3 \\ \left[E(x_{ii}^j) \right] + 1 & E(x_{ii}^j) > 0.3 \end{cases} \qquad \forall i, j \qquad (1)$$

$$pi_i^j = I_i^{j-1} - B_i^{j-1} \qquad \forall i, j \qquad (2)$$

$$S_i^j = pi_i^j + Q_i^j \qquad i = 1, 2, \forall j \qquad (3)$$

$$N_i^j = S_i^j - n_{bundling}^j + n_{unbundling}^j \qquad i = 1, 2, \forall j \qquad (4)$$

$$N_b^j = pi_b^j + n_{bundling}^j - n_{unbundling}^j \qquad \forall j \qquad (5)$$

$$Q_1^j \ge \left| \min\{0, pi_1^j\} \right| + \left| \min\{0, pi_b^j\} \right| - \max\{0, pi_1^j\} - \max\{0, pi_b^j\} \qquad \forall j \qquad (6)$$

$$Q_2^j \ge \left| \min\{0, pi_2^j\} \right| + \left| \min\{0, pi_b^j\} \right| - \max\{0, pi_2^j\} - \max\{0, pi_b^j\} \qquad \forall j \qquad (7)$$

$$n_{bundling}^j \ge \left| \min\{0, pi_b^j\} \right| \qquad \forall j \qquad (8)$$

$$n_{bundling}^j \le \max\left\{0, \min\{S_1^j, S_2^j\}\right\} \qquad \forall j \qquad (9)$$

$$n_{unbundling}^j \ge \left| \min\{0, S_1^j, S_2^j\} \right| \qquad \forall j \qquad (10)$$

$$n_{unbundling}^j \le \max\{0, pi_b^j\} \qquad \forall j \qquad (11)$$

$$n_{bundling}^j \times n_{unbundling}^j = 0 \qquad \forall j \qquad (12)$$

$$I_i^j, N_i^j \ge 0, \quad n_{bundling}^j, n_{unbundling}^j, Q_i^j, B_i^j \in \{0, 1, 2, ...\}, \quad pi_i^j, S_i^j \text{ free} \qquad \forall i, j \qquad (13)$$

Constraints (1) modeled the case in which customers wait until the next period. The potential and realized inventories of products are calculated with respect to levels of their inventories, backorders, and order quantities in (2) and (3), respectively. Effects of bundling/ unbundling decisions on the inventory levels of products are determined in (4) and (5). Order quantities of Products 1 and 2 are determined in (6) and (7), respectively, with respect to the potential inventory levels of Products 1, 2, and the bundle. Constraints (8) and (9) determine numbers of products to be bundled. Also, unbundling numbers of products are determined using Constraints (10) and (11). Constraints (12) declare that only one of bundling and unbundling is done in every period. Finally, decision variables are defined in (13).

NUMERICAL EXAMPLE

To show validity of the proposed bi-product stochastic dynamic program, a numerical example is considered, for which the data presented in Table 1 are utilized.

In the numerical example, a two period planning horizon is considered for which different conditions (cases) of the problem is calculated and for every case, optimized values of the second period are determined. The obtained results are shown in Table 2.

Table 1: Value of implemented parameters in the numerical example

$\mu_1 = 10$	$ch_1 = 0.25$	$c_2 = 3.5$	$cb_2 = 0.5$	$b_2 = 2$
$\mu_2 = 10$	$ch_2 = 0.25$	$c_{bundling} = 9$	$cb_b = 0.5$	$b_b = 4$
$\sigma_1 = \sigma_2 = 1$	$ch_b = 0.5$	$c_{unbundling} = 0.75$	$\lambda = 3$	$B_1^0 = B_2^0 = B_b^0 = 0$
$\rho = -0.9$	$c_1 = 3.5$	$cb_1 = 0.5$	$b_1 = 2$	$I_1^0 = I_2^0 = I_b^0 = 0$

Table 2: Results of optimization for the second period, by considering each case of the first period

Case	n_1	n_2	n_b	$P(n_1,n_2,n_b,p_i)$	B_1^1	B_2^1	B_b^1	Q_1^2	Q_2^2	$p_1^2=p_2^2$	p_b^2	$n_{bundling}^2$	$n_{unbundling}^2$	$Revenue^2$
1	0	0	0	0.062	0	0	0	0	0	12	15	1	0	44.51
2	0	1	0	0.086	0	0	0	0	1	12	15	1	0	41.01
3	0	2	0	0.096	0	0	0	0	2	12	15	1	0	37.51
4	1	0	0	0.086	0	0	0	1	0	12	15	1	0	41.01
5	1	1	0	0.120	0	0	0	1	1	12	15	1	0	37.51
6	1	2	0	0.138	0	0	0	1	2	12	15	1	0	34.01
7	2	0	0	0.096	0	0	0	2	0	12	15	1	0	37.51
8	2	1	0	0.138	0	0	0	2	1	12	15	1	0	34.01
9	2	2	0	0.141	0	0	0	2	2	12	15	1	0	30.51

Table 3: Summary of the obtained results in the considered example

Q_1^1	Q_2^1	p_1^1	p_2^1	p_b^1	$n_{bundling}^1$	$n_{unbundling}^1$	$Revenue^1$	$E(Q_1^2)$	$E(Q_1^2)$	$E(p_1^2)=E(p_2^2)$	$E(p_b^2)$	$E(n_{bundling}^2)$	$E(n_{unbundling}^2)$	$E(Revenue^2)$
2	2	10	10	21	0	0	41.776	1.1	1.1	11.57	14.46	0.96	0	35.4

It is noted that the sum of all probabilities of the problem cases does not equal one, because probabilities of the cases in which backorder is occurred for both products are assumed zero. However, summary of the results are presented in Table 3.

CONCLUSION AND FUTURE RESEARCH DIRECTIONS

Bundle pricing is one of the most promising marketing strategies which have been successfully adopted by diverse fields of industry and services. Although there are a number of published works in this field, which have focused on different aspects of the problem, only handful instances are devoted to the optimizing prices of the products and their bundles. In this regard, this paper addressed pricing problem of a retailer who offered two products and their bundle to her coming customers. In the considered problem, customers were received upon a Poisson distribution with demands following bi-variate distribution function and they might buy either individual products or their bundle, or leave without any products. Also, it is assumed that shortage might occur solely for one of the products. In this case, customer might switch to the other product, or wait until the next period with a backorder cost charged to the retailer, or leave without no purchase (a lost sale cost is

charged). To tackle the problem, a stochastic dynamic programming is proposed with an objective function of retailer's net profit maximization. Finally, a numerical example was developed to validate the proposed model. The developed example was solved for a two-period planning horizon. The results were obtained for the different conditions of the problem. To continue research direction of this paper, two issues are considered. First, it is highly recommended to integrate the proposed dynamic program with a heuristic procedure to distinguish more profitable states of any stage to search solution space of the problem more efficiently. Also, it might be impressive to adopt a metaheuristic algorithm to enhance convergence of the developed dynamic program.

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