Prijemni ispit iz Matematike

402. (1) Rešiti jednačinu

$$x^2 + 3\alpha x + \alpha^2 = 0,$$

gde je α parametar.
403. (2) Rešiti jednačinu

$$\log_{x-1}(3x-5) = 2.$$

404. (3) Uprostiti izraz

$$\left[\frac{(a+b)^2}{a^2+ab+b^2} - \frac{a^3-ab^2}{a^3-b^3}\right] \cdot \left[\frac{a+b}{b} - \frac{b}{a+b}\right] + \frac{b^2-ab}{a^2+ab+b^2}.$$

405. (4) Rešiti jednačinu

$$2 \cdot 4^x - 17 \cdot 2^x + 8 = 0.$$

406. (5) Ako je

$$g(x) = \frac{x+1}{x-1}, \qquad f(x) = \frac{x+2022}{x-2022},$$

izračunati $g(f(x))$.

Prijemi ispit iz matematike

407. (1) Uprostiti izraz

$$\frac{x^2-xy+y^2}{x^2-y^2} \cdot \left(\frac{x-y}{x+y} - \frac{x^3-y^3}{x^3+y^3}\right).$$

408. (2) Rešiti jednačinu

$$4^x - 10 \cdot 2^{x-1} = 24.$$

409. (3) Rešiti jednačinu

$$\log_{x-2}(2x-1) = 2.$$

Prijemi ispit iz matematike IIZZS

410. (1) Dokazati

$$\left[\left(\frac{4}{5} + \frac{3}{2} : \frac{1}{6}\right) \cdot \left(8 + \frac{9}{7}\right)\right]^{-1} = \frac{1}{91}.$$

411. (2) Uprostiti izraz

$$\left(a - \frac{1}{a}\right) \cdot \left(a - \frac{16}{a}\right) : \left(\frac{a}{4} - \frac{4}{a}\right).$$

412. (3) Rešiti jednačinu

$$\log_3(x^2 - 6x + 14) = 2.$$

REŠENJA

2022.

402. (1)

$$x_{1,2} = \frac{-3\alpha \pm \sqrt{9\alpha^2 - 4\alpha^2}}{2} = \frac{-3\alpha \pm \sqrt{5}|\alpha|}{2} = \frac{-3\alpha \pm \sqrt{5}\alpha}{2}$$

$$x_1 = \frac{\alpha(-3-\sqrt{5})}{2}, \qquad = \frac{\alpha(-3+\sqrt{5})}{2}.$$

403. (2) Jednačina ima smisla za $x-1>0$, $x-1 \neq 1$, $3x-5>0$, odnosno $x \in (\frac{5}{3}, 2) \cup (2, \infty)$. Tada je

$$(x-1)^2 = 3x-5, \qquad x^2-2x+1 = 3x-5, \qquad x^2-5x+6 = 0, \quad (x-2)(x-3) = 0,$$

$$x_1 = 2, \quad x_2 = 3.$$

tako da je rešenje polazne jednačine $x = 3$.
404. (3)

$$\begin{aligned} I(a,b) &= \left[\frac{(a+b)^2}{a^2+ab+b^2} - \frac{a(a-b)(a+b)}{(a-b)(a^2+ab+b^2)}\right] \cdot \frac{(a+b)^2-b^2}{b(a+b)} + \frac{b^2-ab}{a^2+ab+b^2} \\ &= \frac{(a+b)b}{a^2+ab+b^2} \cdot \frac{a^2+2ab}{b(a+b)} + \frac{b^2-ab}{a^2+ab+b^2} = 1. \end{aligned}$$

405. (4) Ako se uvede smena $2^x = t$ dobija se

$$2t^2 - 17t + 8 = 0, \qquad t_{1,2} = \frac{17 \pm \sqrt{289-64}}{4} = \frac{17 \pm 15}{4},$$

$t_1 = \frac{1}{2}$, $t_2 = 8$, $x_1 = -1$, $x_2 = 3$.
406. (5)

$$g(f(x)) = \frac{\frac{x+2022}{x-2022}+1}{\frac{x+2022}{x-2022}-1} = \frac{x+2022+x-2022}{x+2022-x+2022} = \frac{2x}{2\cdot 2022} = \frac{x}{2022}.$$

Prijemi ispit iz matematike

407. (1)

$$\begin{aligned} I &= \frac{x^2-xy+y^2}{x^2-y^2} \cdot \left(\frac{x-y}{x+y} - \frac{x^3-y^3}{(x+y)(x^2-xy+y^2)}\right) \\ &= \frac{x^2-xy+y^2}{x^2-y^2} \cdot \frac{(x-y)(x^2-xy+y^2)-(x^3-y^3)}{(x+y)(x^2-xy+y^2)} \\ &= \frac{x^3-x^2y+xy^2-x^2y+xy^2-y^3-x^3+y^3}{(x-y)(x+y)^2} \\ &= \frac{2xy^2-2x^2y}{(x-y)(x+y)^2} = \frac{-2xy(x-y)}{(x-y)(x+y)^2} = \frac{-2xy}{(x+y)^2}. \end{aligned}$$

408. (2)

$2^x = t$, $\quad t^2 - 5t - 24 = 0$, $\quad (t+3)(t-8) = 0$, $\quad t_1 = -3$, $\quad t_2 = 8$, $\quad 2^x = 8$, $\quad x = 3$.

409. (3) Jednačina ima smisla za $x-2>0$, $x-2\neq 1$, $2x-1>0$, odnosno $x\in(2,3)\cup(3,\infty)$. Tada je

$$(x-2)^2=2x-1,\qquad x^2-6x+5=0,\qquad x_1=1,\quad x_2=5,$$

tako da je rešenje polazne jednačine $x=5$.

- Matematika sa proverom sklonosti

410. (1)

$$\left[\left(\frac{4}{5}+\frac{9}{1}\right)\cdot\frac{65}{7}\right]^{-1}=\left[\frac{49}{5}\cdot\frac{65}{7}\right]^{-1}=[7\cdot 13]^{-1}=\frac{1}{91}.$$

411. (2)

$$\frac{a^2-1}{a}\cdot\frac{a^2-16}{a}\cdot\frac{4a}{a^2-16}=\frac{4(a^2-1)}{a}.$$

412. (3)

$$x^2-6x+14=9,\qquad x^2-6x+5=0,\qquad (x-1)(x-5)=0,\qquad x_1=1,\quad x_2=5.$$