

Prijemi ispit iz matematike jun 2020

- (1) Dokazati da je za $m > n > 0$

$$\left(\frac{m + \sqrt{m^2 - n^2}}{m - \sqrt{m^2 - n^2}} - \frac{m - \sqrt{m^2 - n^2}}{m + \sqrt{m^2 - n^2}} \right) \cdot \frac{n^2}{4m\sqrt{m^2 - n^2}} = 1.$$

- (2) Rešiti jednačinu

$$\left(\frac{5}{2}\right)^{x^2} + \left(\frac{2}{5}\right)^{x^2} = \frac{29}{10}.$$

- (3) Rešiti jednačinu

$$\log_{x^3} 8 - \log_{\frac{1}{x^2}} 2 = 3.$$

- (4) Pokazati tačnost jednakosti

$$\frac{1 + \sin \alpha + \cos \alpha}{1 + \sin \alpha - \cos \alpha} = \cot \frac{\alpha}{2}.$$

Uputstvo. Npr. $\sin \alpha = 2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2}$, $\cos \alpha = \cos^2 \frac{\alpha}{2} - \sin^2 \frac{\alpha}{2}$.

Prijemi ispit iz matematike jun 2020 - Zaštita

- (1) Uprostiti izraz

$$\left(\frac{3}{2} : \frac{3}{4} + 0,5 \cdot \frac{2}{3}\right) \cdot \frac{3}{7}.$$

- (2) Dokazati jednakost

$$\frac{(x+2)^2 - 4x}{x^2 - 4} \cdot \frac{2x+4}{x^2+4} = \frac{2}{x-2}.$$

- (3) Rešiti jednačinu

$$\log_2(x^2 - 5x + 10) = 2.$$

Rešenja

JUN

1.
$$\frac{(m + \sqrt{m^2 - n^2})^2 - (m - \sqrt{m^2 - n^2})^2}{(m - \sqrt{m^2 - n^2}) \cdot (m + \sqrt{m^2 - n^2})} \cdot \frac{n^2}{4m\sqrt{m^2 - n^2}} =$$

$$\frac{4m\sqrt{m^2 - n^2}}{m^2 - (m^2 - n^2)} \cdot \frac{n^2}{4m\sqrt{m^2 - n^2}} = 1.$$

2. Smena $t = \left(\frac{5}{2}\right)^{x^2}$ daje $t + \frac{1}{t} = \frac{29}{10}$, $10t^2 - 29t + 10 = 0$, $t_{1,2} = \frac{29 \pm \sqrt{441}}{20} =$
 $\frac{29 \pm 21}{20}$, $t_1 = \frac{5}{2}$, $t_2 = \frac{5}{2}$, $x_1 = 1$, $x_2 = -1$.

3.

$$\frac{3}{3} \log_x 2 + \frac{1}{2} \log_x 2 = 3, \quad \frac{3}{2} \log_x 2 = 3, \quad \log_x 2 = 2, \quad x^2 = 2, \quad x_1 = \sqrt{2}.$$

Vrednost $x_2 = -\sqrt{2}$ se odbacuje jer u polaznoj jednačini mora biti $x > 0$.

4.

$$\begin{aligned} \frac{1 + \sin \alpha + \cos \alpha}{1 + \sin \alpha - \cos \alpha} &= \frac{1 + 2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2} + \cos^2 \frac{\alpha}{2} - \sin^2 \frac{\alpha}{2}}{1 + 2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2} - \cos^2 \frac{\alpha}{2} + \sin^2 \frac{\alpha}{2}} \\ &= \frac{\cos^2 \frac{\alpha}{2} + 2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2} + \sin^2 \frac{\alpha}{2}}{\sin^2 \frac{\alpha}{2} + 2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2} + \sin^2 \frac{\alpha}{2}} \\ &= \frac{2 \cos \frac{\alpha}{2} \left(\cos \frac{\alpha}{2} + \sin \frac{\alpha}{2} \right)}{2 \sin \frac{\alpha}{2} \left(\cos \frac{\alpha}{2} + \sin \frac{\alpha}{2} \right)} \\ &= \frac{\cos \frac{\alpha}{2}}{\sin \frac{\alpha}{2}} = \cot \frac{\alpha}{2}. \end{aligned}$$

ZASTITA JUN

1.

$$\left(2 + \frac{1}{3}\right) \frac{3}{7} = 1.$$

2. -

3.

$$x^2 - 5x + 6 = 0, \quad x_{1,2} = 2, 3.$$